BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR Secondary Co-Ed DAY CUM BOYS' RESIDENTIAL SCHOOL

Mid Term Examination- 2024

MATHEMATICS (041)

Class: XII Science — Duration: 3 Hour
Date: 16/09/24 — MARKING SCHEME — Max. Marks: 80

Q No.	Answer	Scheme
1	**(D)**	$\{2,3,4,5\}$
2	**(D)**	Not defined
3	**(B)**	$0< y < \frac{\pi}{2}$ or $\frac{\pi}{2} < y < \pi$
4	**(B)**	$\frac{2\pi}{3}$
5	(A)	Square matrix
6	**(A)**	**6**
7	**(B)**	**2**
8	**(D)**	**9**
9	**(B)**	± 3
10	**(C)**	**4**
11	**(B)**	$\frac{7}{2}$
12	**(A)**	**2**
13	**(B)**	$-e^{-x}$
14	**(B)**	4π
15	**(D)**	**no value of x**
16	**(C)**	**1**
17	**(D)**	$\frac{-2}{x} + c$
18	**(A)**	$\frac{(3x+4)^4}{12} + c$
19	**(A)**	Both A and R are true and R is the correct explanation of A.
20	**(D)**	A is false but R is true.
21	Domain $=\{0, \pm 3, \pm 4, \pm 5\}$.	
22	Since, cos is negative in second quadrant, Principal value is $(\pi - \frac{\pi}{4}) = \frac{3\pi}{4}$ **OR** $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$ and $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$	

	$\frac{\pi}{3}$ +2 $\frac{\pi}{6}$ = $\frac{2\pi}{3}$
23	$\frac{dx}{d\theta} = a(1 + cos\theta\ )$ $\frac{dy}{d\theta} = asin\theta$ $\frac{dy}{dx} = \frac{asin\theta}{a(1+cos\theta)}$= tan$\frac{\theta}{2}$.
24	Let r be the radius and A be the area of circle. A= πr^2 $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ = $2\pi x 7 x 0.5$ $= 7\pi\ cm^2$ **OR** Marginal cost (MC) = $\frac{d(C(x))}{dx}$ = $\frac{d}{dx}$ ($0.05x^3$ -0.02x^2 +30x +50000) = 0.0153x^2 -0.04x +30 = 0.135-0.12+30 = 30.015 Cost is Rs. 30.02
25	Let 1+2x^2 =t, diff b/s w.r.t x, 4xdx=dt, dx = 1/4dt $\frac{1}{4}\int\frac{1}{t}dt$ = $\frac{1}{4}$log(t) +c $\frac{1}{4}$log (1+2x^2) +c
26	We have f(x) =x+1 $\forall\ x \in R$ For any y$\in R, x = y - 1$ f(y-1) = y-1+1 = y Hence, f is onto
27	We have, A= $\begin{bmatrix}2 & 4 & 0\\3 & 9 & 6\end{bmatrix}$ and B= $\begin{bmatrix}1 & 4\\2 & 8\\1 & 3\end{bmatrix}$ Now, AB = $\begin{bmatrix}10 & 40\\27 & 102\end{bmatrix}$ (AB)' = $\begin{bmatrix}10 & 27\\40 & 102\end{bmatrix}$ Also, B' = $\begin{bmatrix}1 & 2 & 1\\4 & 8 & 3\end{bmatrix}$, A' = $\begin{bmatrix}2 & 3\\4 & 9\\0 & 6\end{bmatrix}$ B'A' = $\begin{bmatrix}10 & 27\\40 & 102\end{bmatrix}$ Hence, (AB)' = B'A' **OR** We have , A= $\begin{bmatrix}p & q\\r & s\end{bmatrix}$, A' = $\begin{bmatrix}p & r\\q & s\end{bmatrix}$ A+A' = $\begin{bmatrix}2p & q+r\\q+r & 2s\end{bmatrix}$ ½(A+A') = $\begin{bmatrix}p & \frac{q+r}{2}\\\frac{q+r}{2} & s\end{bmatrix}$ Also A- A' = $\begin{bmatrix}0 & q-r\\q-r & 0\end{bmatrix}$ = ½ (A-A') = $\begin{bmatrix}0 & \frac{q-r}{2}\\\frac{q-r}{2} & 0\end{bmatrix}$

	A= ½ (A+A') +1/2 (A-A') = $\begin{bmatrix} 2p & q+r \\ q+r & 2s \end{bmatrix} + \begin{bmatrix} 0 & \frac{q-r}{2} \\ \frac{q-r}{2} & 0 \end{bmatrix}$
28	Let $\|A\|$= 10, A^{-1} exists $A^{-1} = \frac{1}{\|A\|}$ adjA = $\frac{1}{10}\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}$ Now, AX=B , A^{-1}(AX)= A^{-1}B , X= A^{-1}B X= $\frac{1}{10}\begin{bmatrix} -2 & 4 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$ X= $\begin{bmatrix} 6 & 2 \\ \frac{11}{2} & 2 \end{bmatrix}$
29	Since f(x) is continuous at every point of its domain $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$ $\lim_{h\to0^-}[5(1-h)-4] = \lim_{h\to0^+}[4(1+h)^2+3b(1+h)] = 5-4$ 1 =4+3b 3b=-1 b= -1 **OR** Since , it's given that f is continuous on [0,8] F(x) is continuous at x=2 and x=4 Therefore, LHL =f(x) = RHL At x=2, $\lim_{x\to2^-} f(x) = f(2)$ 2a+b=4 At x=4, $\lim_{x\to4^+} f(x) = f(4)$ 8a+5b =14 On solving we get a= 3, b=12
30	f(x) = tanx -4x , f'(x) = $\sec^2$x -4 f'(x) < 0 for $-\frac{\pi}{3} < x < \frac{\pi}{3}$ Therefore, f(x) is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$
31	We write, x+2 = A$\left[\frac{d}{dx}(2x^2+6x+5)\right]$ +B We get, A= ¼, B= ½ I = ¼ $\int \frac{4x+6}{2x^2+6x+5}$dx +1/2 $\int \frac{dx}{2x^2+6x+5}$ = ¼ log($2x^2$+6x+5) + ½ $\tan^{-1}$(2x+3) +c
32	Z is the set of all integers and R=$\{(a,b): a,b \in Z \ and \ a+b \ is \ even\}$ Reflexive: (a+a) is even, hence R is Reflexive Symmetric: a+b is even, then b+a is also even, hence r is symmetric Transitive: a+b is even and b+c is even, then a+c is also even, hence R is transitive. Since, R is Reflexive, Symmetric and transitive therefore R is Equivalence relation.
33	The given system of equation can be written as $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$, AX = B. $\|A\|$ =18, A^{-1} exists.

	$\therefore X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$ $= \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$
34	Let y= $(sinx)^{cosx}$ log y = cosx logsinx Differentiate w,r.t.x $\frac{dy}{dx} = y\left[\frac{cos^2x}{sinx}\ sinxlog(sinx)\right]$ $= (sin)^{cosx}\left[\frac{cos^2x}{sinx}\ sinxlog(sinx)\right]$ **OR** Let y= $(x+1)^2(x+2)^3(x+3)^4$ logy = 2log(x+1) + 3log(x+2) +4log(x+3) Differentiate w.r.t x $\frac{1}{y}\frac{dy}{dx} = \left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$ $\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$ $\frac{dy}{dx} = (x+1)^2(x+2)^3(x+3)^4\left[\frac{2}{x+1} + \frac{3}{x+2} + \frac{4}{x+3}\right]$ $= (x+1)(x+2)^2(x+3)^3(9x^2 + 34x + 29)$
35	We have, f(x) = $-\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$ $f'(x) = -3x^3-24x^2-45x = -3x(x^2 + 8x + 15)$ $f'(x) = 0 = -3x(x^2 + 8x + 15)$ x=0, x= -3, x=-5 are the possible point for local maxima and minima. Now, $f''(x) = -9x^2-48x-45$ At x=0, is point of local maxima, At x= -3, is a point of local minima At x= -5, is a point of local maxima. Local maximum value is $\frac{295}{4}$. **OR** Let one number is x and other number is 64-x $f(x)= x^3 + (64-x)^3$ $f'(x) = 3x^2 + 3(64-x)^2 (-1)$ for f'(x) =0 x=32 now, f''(x) = 384 $>$ 0at x=32 Thus, f(x) is minimum when 64 divided in two equal parts.
36	i) $\alpha = 30°$ ii) $\beta = 60°$ iii) $\angle ABC = 90°$

37	By using the concept of adjoint and inverse of matrix, we will have i) Rs1 ii) Rs 5 iii) Rs. 2
38	i) $(2x^2 + 8xy)m^2$ ii) $x^2y = 32$ iii) Half of its width or length.

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